Black Hole Evaporation and Complementarity*

summary of lectures presented by Erik Verlinde†

About twenty years ago Hawking made the remarkable suggestion that the black hole evaporation process will inevitably lead to a fundamental loss of quantum coherence [1]. The mechanism by which the quantum radiation is emitted appears to be insensitive to the detailed history of the black hole, and thus it seems that most of the initial information is lost for an outside observer. However, direct examination of Hawking's original derivation (or any later one) of the black hole emission spectrum shows that one inevitably needs to make reference to particle waves that have arbitrarily high frequency near the horizon as measured in the reference frame of the in-falling matter. This exponential red-shift effect associated with the black hole horizon leads to a breakdown of the usual separation of length scales (see e.g. [2] and [3]), and effectively works as a magnifying glass that makes the consequences of the short distance, or rather, high energy physics near the horizon visible at larger scales to an asymptotic observer.

Let us begin by reviewing the derivation of Hawking radiation in the s-wave sector, while at first ignoring the effects due to gravitational back-reaction. We introduce two null-coordinates u and v such that at a large distance $r \to \infty$ we have $u \to t + r$ and $v \to t - r$. Following [1], we imagine sending a small test-particle backwards in time from future null infinity $\mathcal{I}^+$ and letting it propagate all the way through to $\mathcal{I}^-$ (see figure 1.). To relate the form of this signal in the two asymptotic regions, we use the wave equation on the *fixed* background geometry of the collapsing black hole.

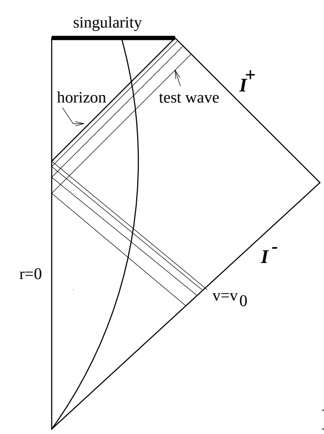

Fig. 1

*based on work with Y. Kiem, K. Schoutens and H. Verlinde [4, 5]. This is a modified version of lecture notes that will appear in the proceedings of the Trieste Spring School of 1993.

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From the condition that the field is regular at the origin $r = 0$ one deduces that the outgoing s-wave ϕ_{out} of the massless scalar field ϕ and the corresponding in-coming wave ϕ_{in} are related by a reparametrization

$$\phi_{in}(v) = \phi_{out}(u(v)), \tag{1}$$

$$u(v) = v - 4M \log[(v_0 - v)/4M], \tag{2}$$

where M denotes the black hole mass and v_0 the critical in-going time, *i.e.* the location of the in-going light-ray that later will coincide with the black hole horizon. Thus an outgoing s-wave with a given frequency ω translates to an in-signal

$$e^{i\omega u(v)} = e^{i\omega v}(\frac{v_0 - v}{4M})^{-i4M\omega}\theta(v_0 - v), \tag{3}$$

that decomposes as a linear superposition of incoming waves with very different frequencies. The out-going modes b_ω (*i.e.* the Fourier coefficients of ϕ_{out}) are therefore related to the in-coming modes a_ξ via a non-trivial Bogoljubov transformation of the form

$$b_\omega = \sum_\xi \alpha_{\omega\xi} a_\xi + \sum_\xi \beta_{\omega\xi} a^\dagger_\xi \quad ; \qquad b^\dagger_\omega = \sum_\xi \beta^*_{\omega\xi} a_\xi + \sum_\xi \alpha^*_{\omega\xi} a^\dagger_\xi. \tag{4}$$

where $\alpha_{\omega\xi}$ and $\beta_{\omega\xi}$ are given by the Fourier-coefficients of (3), for example

$$\alpha_{\omega\xi} = \, e^{-i(\xi-\omega)v_0} \, \frac{e^{2\pi M\omega}\Gamma(1 - i4M\omega)}{2\pi\sqrt{\omega\xi}}, \tag{5}$$

The transformation (4) is not invertible, and as a consequence pure in-states $|in\rangle$ are mapped onto *mixed out*-states. In particular, the in-vacuum $|0\rangle$ evolves into the famous Hawking state ρ_H, describing a constant flux of thermal radiation at the Hawking temperature $T_H = \frac{1}{8\pi M}$.

In the above description the incoming particles described by $\phi_{in}(v)$ with $v > v_0$ and the outgoing particles represented by $\phi_{out}(u)$ form independent sectors of the Hilbert space, because the corresponding field operators commute with each other. The underlying classical intuition is that the fields $\phi_{in}(v)$ with $v > v_0$ will propagate into the region behind the black hole horizon, and thus become unobservable from the out-side. However, here we have ignored the fact that the in-falling particles in fact *do* interact with the out-going radiation, because they slightly change the black hole geometry. Furthermore, it can be seen

from (3) that a generic out-going wave carries a diverging stress-energy near the horizon. This indicates that gravitational interactions may indeed be important.

Consider a spherical shell of matter with energy δM that falls in to the black hole at some late time v_1. At this time the Schwarzschild radius slightly increases with an amount $2\delta M$, and as a consequence the time v_0 at which the horizon forms will also change very slightly (see figure 2). A simple calculation shows that

$$\delta v_0 = -c\,\delta M e^{-(v_1 - v_0)/4M}, \tag{6}$$

where, c is a constant of order one. At first it seems reasonable to ignore this effect as long as the change δM is much smaller than M, but, it turns out that, due to the diverging red-shift, the variation in v_0, although very small, has an enormous effect on the wave-function $\phi_{out}(u)$ of an out-going particle.

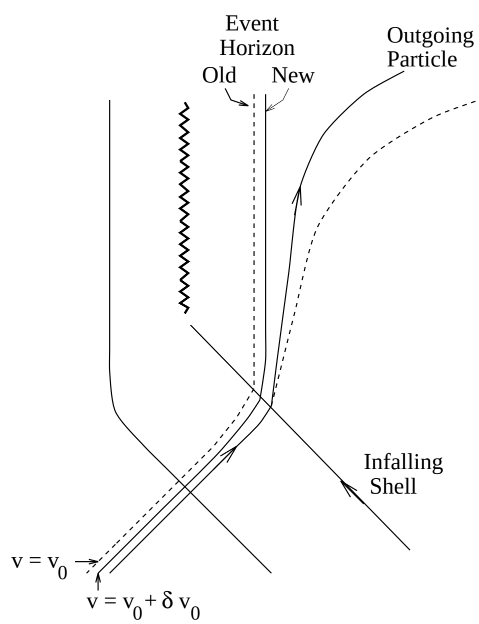

Fig 2.

By combining (1), (2) and (6) one easily verifies that as a result of the in-falling shell, the outgoing particle-wave is delayed by an amount that grows rapidly as a function of u

$$\phi_{out}(u) \to \phi_{out}(u - 4M \log(1 - c\,\frac{\delta M}{4M} e^{(u-v_1)/4M})). \tag{7}$$

Notice that even for a very small perturbation δM the argument of the field ϕ_{out} goes to infinity after a finite time $u_{lim} - u_1 \sim -4M \log(\delta M/M)$. The physical interpretation of this fact is that a matter-particle that is on its way to reach the asymptotic observer at some

time $u > u_{lim}$ will, as a result of the additional in-falling shell, cross the event-horizon and be trapped inside the black-hole horizon.

To take the effect (7) into account in the quantum theory, we divide up the in-falling matter in a classical piece plus a small quantum part that is described in terms of a quantum field $\phi_{in}(v)$. Of course, v_0 is mainly determined by the classical in-falling matter, but in addition there is a small contribution that depends on the fields $\phi_{in}(v)$. For simplicity we assume here that the only effect is that the mass and thus the Schwarschild radius changes as in (6). In this way we find[‡]

$$v_0 = v_0^{cl} - c \int_{v_0^{cl}}^{\infty} dv \, e^{(v_0^{cl}-v)/4M} T_{in}(v) \tag{8}$$

where $T_{in}(v)$ denotes the stress-energy tensor of the $\phi_{in}(v)$ with support $v > v_0^{cl}$.

With this correction we now re-calculate the commutator of the outgoing field $\phi_{out}(u)$ for late times with the in-coming field $\phi_{in}(v)$ for $v > v_0^{cl}$. For this calculation we take the same relation (1), that formed the starting point of Hawking's derivation, but we include in the reparametrization (2) the seemingly negligible quantum contribution in v_0. From the fact that the stress-tensor generates coordinate transformations we deduce

$$[v_0, \phi_{in}(v)] = -ic \, \exp((v_0^{cl} - v)/4M) \partial_v \phi_{in}(v), \tag{9}$$

or equivalently

$$e^{i\xi v_0} \phi_{in}(v) e^{-i\xi v_0} = \phi_{in}(v - 4M \log(1 - \frac{c}{4M} \xi e^{(v_0^{cl}-v)/4M})), \tag{10}$$

Combining this result with (1) and (2) we obtain the following algebra for the *in*$-$ and *out*-fields

$$\phi_{out}(u)\phi_{in}(v) = \exp(ic \, e^{(u-v)/4M} \partial_u \partial_v) \phi_{in}(v) \phi_{out}(u), \tag{11}$$

which is valid at for $v > v_0^{cl}$. The 'exchange algebra' (11) is the quantum implementation of the gravitational back-reaction (7) of the in-falling matter on the out-going radiation. Notice that this algebra is non-local, and, furthermore, that the non-locality grows exponentially with time.

These gravitational interactions lead to a modification of the quantum state of the out-going radiation. To describe this modification we introduce addiditional modes c_ω

[‡]For a detailed discussion of the angular dependence we refer to [5].

describing the in-falling particles at the horizon. In terms of these the original state computed by Hawking can be written as:

$$|\psi\rangle_{Hawking} = \frac{1}{N}\exp\Big\{\int_0^\infty d\omega\, e^{-4\pi M\omega}\, b^\dagger_\omega\, c^\dagger_\omega\Big\}\, |0\rangle_b \otimes |0\rangle_c \tag{12}$$

But when one takes into account the correction (8) due to the gravitational backreaction one obtains for final state

$$|\psi\rangle_{final} = \mathcal{U}\, |\psi\rangle_{Hawking} \tag{13}$$

where

$$\mathcal{U} = \mathrm{T}\exp\Big[\, i \iint_{v_0} du dv\, e^{(u-v)/4M} T_{uu}(u) T_{vv}(v)\, \Big] \tag{14}$$

Notice again the interaction Hamiltonian wich appears in $\mathcal{U}$ contains a pre-factor that grows exponentially with time. This makes clear that these interaction become non-neglible at practically the same time when the black hole radiation starts to appear.

We want to emphasize that in the above derivation we did not make any assumptions other than those already made in the usual derivation of Hawking evaporation. The only extra ingredient that we took into account is the small contribution to v_0 due to the field ϕ. In this sense the relation (11) appears to be unavoidable and independent of which scenario one happens to believe in. Therefore, it is clear that the presence of these large commutators implies that the standard semi-classical picture of the black hole evaporation process needs to be drastically revised. In particular, it tells us that, due to the quantum uncertainty principle, we should be very careful in making simultaneous statements about the in-falling and out-going fields.

Our result supports the physical picture that there is a certain *complementarity* between the physical realities as seen by an asymptotic observer and by an in-falling observer. Indeed, for the latter, the in-falling matter will simply propagate freely without any perturbation, but he (or she) will not see the out-going radiation. For the asymptotic observer, on the other hand, the Hawking radiation is physically real, while the in-falling matter will appear to evaporate completely before it falls into the hole. Finally, we expect that our result may also be of importance in relation with the entropy of black holes. In [6] it was noted that a naive free field calculation of the one-loop correction to the black hole entropy gives an infinite answer. This infinity arises due to the diverging contribution of states that are packed arbitrarily close to the horizon. Our result suggests a possible remedy of this problem, because it shows that in the coordinate system appropriate for this calculation the in- and out-going fields no longer commute when they come very close to the horizon. We believe that this will effectively reduce the number of allowed states, and thereby eliminate the diverging contribution in the entropy calculation.

Acknowledgements.

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